

ASE2010 Applied linear algebra: Homework #2

- 1) *Orthogonal projection.* Let x be an n -vector and u_1, \dots, u_k with $k < n$ be orthonormal n -vectors. The projection of x onto the span of u_1, \dots, u_k is

$$\hat{x} = (u_1^T x)u_1 + \dots + (u_k^T x)u_k \in \mathbb{R}^n$$

and the projection of x onto the span of u_1, \dots, u_{k-1} is

$$\tilde{x} = (u_1^T x)u_1 + \dots + (u_{k-1}^T x)u_{k-1} \in \mathbb{R}^n.$$

Show that \hat{x} is closer to x than \tilde{x} in that

$$\|\hat{x} - x\| \leq \|\tilde{x} - x\|.$$

- 2) *VMLS Exercises.*

- a) **5.1** *Linear independence of stacked vectors.*
- b) **5.4** *Norm of linear combination of orthonormal vectors.*
- c) **5.5** *Orthogonalizing vectors.*
- d) **5.6** *Gram–Schmidt algorithm.*
- e) **5.8** *Early termination of Gram–Schmidt algorithm.*
- f) **6.3** *Block matrix.*
- g) **6.8** *Cash flow to bank account balance.*
- h) **6.10** *Resource requirements.*
- i) **6.12** *Skew-symmetric matrices.*
- j) **6.14** *Norm of matrix-vector product.*
- k) **6.17** *Stacked matrix.*
- l) **6.18** *Vandermonde matrices.*