ASE2010 Applied linear algebra: Homework #3

1) *Rotation matrices.* Consider a matrix A that describes a rotation by θ, that is,

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

a) Explain why $\|y\| = \|x\|$ for any x and θ.

b) Show that the columns of A are orthonormal vectors.

c) Construct a matrix that describes a rotation by $-\theta$?

d) What is A^T? Is it equal to what you obtained from above?

e) Consider a vector x, and suppose that we compute $y = Ax$, and then subsequently compute $z = A^T y$. What is z?

f) What is $A + A^T$? What does it do? Justify your answer by drawing a picture on a plane to illustrate x, Ax, $A^T x$, and $(A + A^T)x$

2) *VMLS Exercises.*

a) **7.1** *Projection on a line.*

b) **7.2** *3-D rotation.*

c) **7.3** *Trimming a vector.*

d) **7.4** *Down-sampling and up-conversion.*