ASE2010 Applied linear algebra: Homework #6

1) *VMLS Exercises.*

   a) **12.1** *Approximating a vector as a multiple of another one.*

   b) **12.4** *Weighted least squares.*

   c) **12.5** *Approximate right inverse.*

   d) **12.8** *Least squares and QR factorization.*

   e) **12.13** *Iterative method for least squares problem.*

2) *Nonnegative least squares.* In this problem, we extend the iterative method from *VMLS Exercise* **12.13** to solve the *nonnegative least squares problem (NNLS)*. We are interested in the following problem.

$$\begin{array}{ll} \underset{x}{\text{minimize}} & \|Ax - b\|^2 \\ \text{subject to} & x \geq 0 \end{array}$$

In other words, we want to find the vector x with nonnegative entries that makes Ax as close to b as possible.

Your kind professor gives you the detailed instructions for solving this. First set up the problem: generate a random 20×10 matrix A and 20-vector b, and also let $\mu = 1/\|A\|^2$.

```
import numpy as np
np.random.seed(2010)
A = np.random.randn(20,10)
b = np.random.randn(20)
mu = 1/np.linalg.norm(A,'fro')**2
```

   a) Set $k = 1$ and randomly generate an initial condition $x^{(1)}$.
   b) Use the following two-step procedures to compute $x^{(k+1)}$ from $x^{(k)}$. Note that the $(x)_+$ operator sets every negative entry of x to zero. For example when $x = (0, 1, -2, 3, 4, -5)$, we have $(x)_+ = (0, 1, 0, 3, 4, 0)$.

$$\begin{aligned} x^{(k+0.5)} &= x^{(k)} - \mu A^T(Ax^{(k)} - b) \\ x^{(k+1)} &= (x^{(k+0.5)})_+ \end{aligned}$$

   c) Terminate the algorithm and return the solution if the amount of the update is sufficiently small ($\|x^{(k+1)} - x^{(k)}\| < 10^{-12}$, for example). Otherwise increase k by 1 and go back to step b).

Run this algorithm once and check if every element in your solution is nonnegative. Fix A, b and repeat this numerical experiments several times with different random initial conditions. Check if every trial converges to the same solution (present appropriate plots and numerical results that explain this).