Large Divert Computational Guidance for Exoatmospheric Vehicles

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***Abstract*—We consider the optimal guidance problems for exoatmospheric vehicles with large divert maneuvers. The large divert impact angle guidance problem was formulated as a nonconvex optimization problem with constraints on the angles between the control vectors and the line-of-sight vectors, and the problem was efficiently solved via first-order methods. A series of numerical simulations against high-speed targets shows that the proposed approach can be successful for a wider range of engagement scenarios compared to the classical impact angle guidance solutions.**

***Index Terms*—large divert guidance, exoatmospheric vehicles, nonconvex optimization, first order methods**

I. Introduction

Exoatmospheric kill vehicles (KV) with divert attitude control systems are popularly used for ballistic missile defense systems. Since the relative velocity between the KV and target is in general very large, forming the head-on interception geometries at the terminal stage is preferable. However, the classical impact angle guidance (IAG) typically requires large acceleration maneuver due to the drastic increase in line-of-sight (LOS) rate in the proximity of the impact, which results in the increase of interception error.

In this paper, a computational guidance approach taking account for these issues is proposed. Rather than formulating the optimal IAG problem via linearization, we directly handle the nonconvex constraints and formulate the optimization problem without linearization. The problem can be efficiently solved via first-order optimization methods. The guidance performance of the proposed approach is compared with that of the classical IAG [1] via a series of numerical simulations.

II. Proposed Guidance Law

A. Problem Formulation

We assume that the KV in this problem has the following characteristics:

- KV's body-1 axis is always aligned to the LOS vector.
- The divert thrust is perpendicular to the LOS vector.
- The magnitude of the divert thrust is limited.

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In other words, KV's attitude control loop is assumed to track the LOS vector so that the target is always visible, and the divert thrust is always perpendicular to the LOS vector.

The IAG for large divert maneuver with the above constraints can be formulated as the following nonconvex optimization problem:

$$\begin{array}{ll} \text{minimize} & \sum_{t=0}^{T-1} ||u_t||^2 \\ \text{subject to} & x_{t+1} = x_t + \Delta t v_t + 0.5\Delta t^2 u_t, \\ & v_{t+1} = v_t + \Delta t u_t, \\ & y_t = p_0 + \Delta t \omega_0 t - x_t, \\ & y_T = 0, \qquad \angle v_T = \theta_f, \\ & ||u_t|| \le u_{\text{ub}}, \qquad u_t^T y_t = 0. \end{array} \tag{1}$$

Here, T is the horizon size and Δt is the sampling interval. The decision variable u_t, x_t and v_t are the KV's control input, position and velocity vectors, respectively. Also, p_0 and ω_0 represent the target's initial position and velocity, and y_t is the KV-to-target LOS vector. The maximum available control is given by u_{ub}. Each constraint represents the KV's dynamics, the LOS vector to the target, interception condition, rhe desired course angle at the predicted impact point, control size limit, and the angle between u_t and y_t.

B. First-order optimization on nonconvex problems

We apply the alternating direction method of multipliers (ADMM) to solve the above nonconvex optimization. The above problem is formulated in the standard ADMM form [2] as below:

$$\begin{array}{ll} \text{minimize} & ||u||^2 + I_{\mathcal{C}_1}(w) + I_{\mathcal{C}_2}(z) \\ \text{subject to} & Pu + q = 0, \\ & Gu + h = z, \qquad w = u. \end{array} \tag{2}$$

where u is the stack of the control input history, P, q, G and h are matrices that relate the dynamics and the course of the KV, and the LOS vector from the KV to the target as well as the desired terminal constraints. The second and third equalities are incorporated to handle the constraints on control vectors, and the constraints on the angle between the control vectors and the LOS vectors. Additional objective terms, $I_{\mathcal{C}_1}(\cdot)$ and $I_{\mathcal{C}_2}(\cdot)$, are the indicator functions that handle the last two

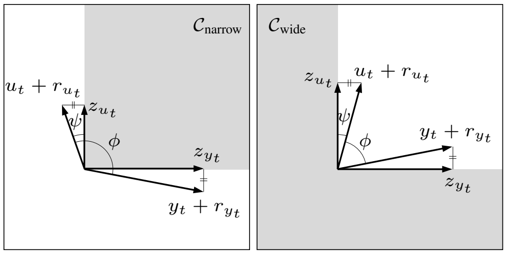

Fig. 1. Projection onto the nonconvex set $\mathcal{C}_2 = \cap_t \left\{ \left(u_t, y_t\right) \mid u_t^T y_t = 0 \right\}$.

constraint sets in the problem (1), $\mathcal{C}_1 = \cap_t \left\{ u_t \mid ||u_t|| \leq u_{\text{ub}} \right\}$, and $\mathcal{C}_2 = \cap_t \left\{ \left(u_t, y_t\right) \mid u_t^T y_t = 0 \right\}$.

Obtaining the solution via the ADMM approach boils down to computing the orthogonal projection of intermediate variables onto the set $\mathcal{C}_1$ and $\mathcal{C}_2$. The intuition behind directly solving the nonconvex problem via the same approach is that the projections onto $\mathcal{C}_1$ and $\mathcal{C}_2$ are easily computable even though $\mathcal{C}_2$ is nonconvex [3].

Projection onto the first (convex) set $\mathcal{C}_1$ is simple: if the argument is outside the ball, it shrinks to the surface.

However, computing the orthogonal projection onto the second (nonconvex) set, $\mathcal{C}_2$, is rather complicated. Fig. 1 displays how the projection on $\mathcal{C}_2$ works. Let ϕ be the angle between $u_t + r_{u_t}$ and $y_t + r_{y_t}$, which are the intermediate control input and LOS vector, respectively. Then the angle ψ is chosen so that the distance between $u_t + r_{u_t}$ and z_{u_t} and the distance between $y_t + r_{y_t}$ and z_{y_t} are equal. If angle ϕ is greater than 90 degrees, $u_t + r_{u_t}$ and $y_t + r_{y_t}$ are projected onto $\mathcal{C}_{\text{narrow}}$, while both vectors are projected onto $\mathcal{C}_{\text{wide}}$ when ϕ is smaller than 90 degrees.

Once these projections are obtained, the standard ADMM update steps to converge the problem can be followed.

III. Simulation Results

The proposed approach was analyzed by a set of numerical simulations on a variety of engagement instances shown in Fig. 2. Initial conditions of the KV are given by $x_0 = \left(0, 0\right)$, $v_0 = \left(0, 10\right)$ and $u_{\text{ub}} = 8$. The target is initially at $d = 100$ units apart along the range direction with the cross-track distance X varying from 20 to 40, and approaches along the course angle η varying from 255 degrees to 285 degrees. The minimum time solution for the problem was searched via bisection on T, and Fig. 3 and Fig. 4 display in detail the simulation results from an engagement instance with $X = 40$ and $\eta = 255$ degrees, where the IAG fails to achieve the desired impact angle θ_f due to the acceleration divergence.

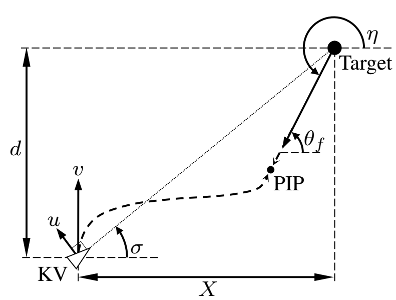

Fig. 2. Engagement geometry.

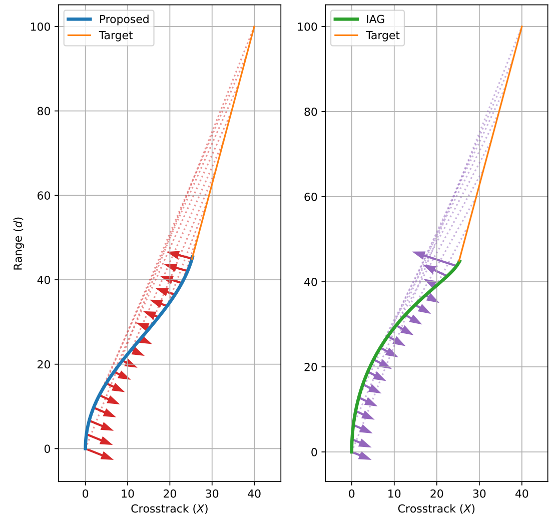

Fig. 3. Engagement trajectories of the proposed guidance law and the IAG. The arrows display the size and the direction of the maneuver accelerations.

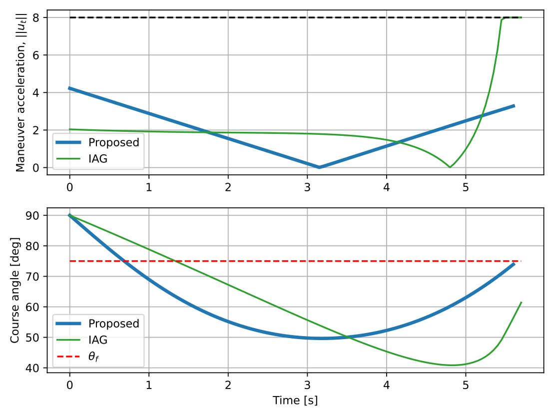

Fig. 4. Maneuver accelerations and course angles.

IV. Conclusion

A nonconvex formulation of the large divert guidance problem was introduced and it was efficiently solved via the first-order optimization method. Numerical analysis on the guidance performance indicated that the proposed approach can be effective for a wider range of engagement scenarios compared to that of the conventional approaches.

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