A first order method with nonconvex projection for optimal powered-descent guidance

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ABSTRACT

This paper introduces a first order method for optimal powered-descent guidance problems that directly handles the nonconvex constraints associated with the maximum and the minimum thrust bounds with mass variations. This issue has been conventionally circumvented via "lossless convexification" that lifts a nonconvex feasible set to a higher dimensional convex set, and via linear approximation of another nonconvex feasible set defined by exponential functions [1–3]. However the solution obtained from this approach sometimes results in an infeasible solution if projected back to the original space, especially when the problem considers a non-minimum time control. We introduce a first order approach that makes use of direct computation of the orthogonal projections onto the nonconvex sets, and we show that the approach provides a feasible solution with better performance even for the non-minimum time cases compared conventional techniques. We claim that the proposed approach presents more flexibility in generating the feasible trajectories for a wide variety of planetary soft landing problems.

PROBLEM FORMULATION

The optimal powered-descent guidance problem we consider in this paper can be formulated as

$$
\begin{array}{ll}
\underset{t_f, T(\cdot)}{\text{minimize}} & \int_0^{t_f} \|T(t)\|\, dt \\
\text{subject to} & \dot{x}(t) = v(t), \\
& \dot{v}(t) = g + T(t)/m(t), \\
& \dot{m}(t) = -\alpha \|T(t)\|, \\
& 0 < T_{\min} \le \|T(t)\| \le T_{\max}, \\
& e_1^T T(t) \ge \|T(t)\| \cos\theta, \\
& m(0) = m_{\text{wet}},\ m(t_f) \ge m_{\text{dry}}, \\
& x(0) = x_{\text{init}},\ v(0) = v_{\text{init}},\ x(t_f) = v(t_f) = 0, \qquad \forall t \in [0, t_f].
\end{array}
\tag{1}
$$

which is equivalent to

$$
\begin{aligned}
\underset{u_k,\sigma_k}{\text{minimize}} \quad & -z_N \\
\text{subject to} \quad & x_{k+1} = x_k + hv_k + 0.5h^2 u_k, \qquad v_{k+1} = v_k + h\,(g + u_k), \\
& z_{k+1} = z_k - \alpha h \sigma_k, \qquad \|u_k\| = \sigma_k, \\
& T_{\min} e^{-z_k} \leq \sigma_k \leq T_{\max} e^{-z_k}, \\
& e_1^T u_k \geq \sigma_k \cos\theta, \qquad z_0 = \log m_{\text{wet}}, \\
& \log\left(m_{\text{wet}} - \alpha T_{\max} kh\right) \leq z_k \leq \log\left(m_{\text{wet}} - \alpha T_{\min} kh\right), \\
& x_0 = x_{\text{init}},\ v_0 = v_{\text{init}},\ x_N = v_N = 0, \qquad \forall\ k = \{0, 1, \cdots N-1\}.
\end{aligned} \tag{2}
$$

if discretized. Note that the constraints associated with the mass change and the thrust bounds are nonconvex.

We solve this problem via the alternating direction method of multipliers (ADMM) handling the nonconvex sets by directly computing the orthogonal projection onto them. The two associated nonconvex sets onto which the orthogonal projection is computed is as follows.

$$
\begin{aligned}
\mathcal{C}_1 &= \left\{(u,\sigma) \in \mathbb{R}^4 \mid \|u\|_2 = \sigma\right\} \\
\mathcal{C}_2 &= \left\{(z,\sigma) \in \mathbb{R}^2 \mid T_{\min} e^{-z} \leq \sigma \leq T_{\max} e^{-z}\right\}
\end{aligned}
$$

The projection onto the surface of the Lorentz cone defined by $\mathcal{C}_1$ is explicitly given by

$$
\Pi_{\mathcal{C}_1}(u,\sigma) = \begin{cases} (u,\sigma) & \text{if } \|u\|_2 = \sigma \\ 0 & \text{if } \sigma \leq -\|u\|_2 \text{ and } \sigma \leq 0 \\ \left(\frac{\|u\|_2 + \sigma}{2\|u\|_2} u, \frac{\|u\|_2 + \sigma}{2}\right) & \text{otherwise} \end{cases} \tag{3}
$$

and the projection onto the nonconvex corridor defined by the exponential curves $\mathcal{C}_2$ can be efficiently computed by numerical methods such as the Newton-Raphson methods.

These operations can be conveniently exploited in the ADMM update steps.

REFERENCES

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