ASE2010 Applied linear algebra: Homework #1

1) *Linear functions.*

   a) Show that an inner product function, $f(x) = a^T x$, is linear.

   b) Show that any scalar-valued linear function $f(x)$ satisfying superposition can be expressed as an inner product function, say $f(x) = a^T x$. Explicitly state the elements of a in terms of f.

2) *Affine functions.*

   a) Show that an inner product function plus a shift, $f(x) = a^T x + b$, is affine.

   b) Show that any scalar-valued affine function $f(x)$ satisfying the restricted superposition (superposition defined for linear combination with coefficients that sum to 1) can be expressed as an inner product function plus a shift, say $f(x) = a^T x + b$. Explicitly state the elements of a and b in terms of f.

3) *Cauchy-Schwarz inequality.* Show that any two vectors $a, b \in \mathbb{R}^n$ satisfy the following. Also state the condition under which the inequality is tight.

$$|a^T b| \leq \|a\|\|b\|.$$

4) *Angle between two vectors.* Show that any two vectors $a, b \in \mathbb{R}^n$ satisfy the following, where θ is the angle between a and b. You may provide a proof for the two-dimension case, which easily generalizes to general n-dimension cases.

$$a^T b = \|a\|\|b\| \cos\theta.$$

5) *Parallelogram.* Draw two different vectors u and v out from the origin. Complete two more sides to make a parallelogram with diagonals $w = u + v$ and $z = u - v$. Show that $\|w\|^2 + \|z\|^2 = 2\|u\|^2 + 2\|v\|^2$.

6) *VMLS Exercises.*

   a) **2.3** *Motion of a mass in response to applied force.*

   b) **2.12** *Price change to maximize profit.*

   c) **3.12** *Nearest point to a line.*