ASE2010 Applied linear algebra: Homework #2

1) *Orthogonal projection.* Let x be an n-vector and $u_1, \dots, u_k$ with $k < n$ be orthonormal n-vectors. The projection of x onto the span of $u_1, \dots, u_k$ is

$$\hat{x} = (u_1^T x)u_1 + \cdots + (u_k^T x)u_k \in \mathbb{R}^n$$

and the projection of x onto the span of $u_1, \dots, u_{k-1}$ is

$$\tilde{x} = (u_1^T x)u_1 + \cdots + (u_{k-1}^T x)u_{k-1} \in \mathbb{R}^n.$$

Show that $\hat{x}$ is closer to x than $\tilde{x}$ in that

$$\|\hat{x} - x\| \leq \|\tilde{x} - x\|.$$

2) *Transformation by orthonormal vectors.* Suppose that the columns of $T \in \mathbb{R}^{n \times k}$ with $k \leq n$ are orthonormal. Show that the transformation $x \mapsto Tx$ satisfies the following properties.

   a) The transformation is *isometric*, *i.e.*, it preserves *distance* between vectors,

   $$\|Tx - Ty\| = \|x - y\|, \quad \forall x, y.$$

   b) The transformation is *conformal*, *i.e.*, it preserves *angle* between vectors,

   $$\angle(Tx, Ty) = \angle(x, y), \quad \forall x, y.$$

3) *VMLS Exercises.*

   a) **5.1** *Linear independence of stacked vectors.*

   b) **5.4** *Norm of linear combination of orthonormal vectors.*

   c) **5.5** *Orthogonalizing vectors.*

   d) **5.6** *Gram–Schmidt algorithm.*

   e) **5.8** *Early termination of Gram–Schmidt algorithm.*

   f) **6.3** *Block matrix.*

   g) **6.8** *Cash flow to bank account balance.*

   h) **6.10** *Resource requirements.*

   i) **6.12** *Skew-symmetric matrices.*

   j) **6.14** *Norm of matrix-vector product.*

   k) **6.17** *Stacked matrix.*

   l) **6.18** *Vandermonde matrices.*