

ASE2010 Applied linear algebra: Homework #4

- 1) *Lower triangular matrices.* Let A be a lower triangular $n \times n$ matrix. Verify the following properties.
- a) If B is a lower triangular $n \times n$ matrix, then the product AB is lower triangular.
 - b) The matrix A^k is lower triangular for all positive integers k .
 - c) If A is invertible, then A^k is lower triangular for all integers k (positive or negative).

- 2) *QR factorization of matrix with orthonormal columns.* Suppose the columns of a matrix A are orthonormal, and we (attempt) to compute its QR factorization $A = QR$. Which of the following must be true?
- a) The QR factorization will fail.
 - b) $R = I$
 - c) $R = A$
 - d) $Q = I$
 - e) $Q = A$

- 3) *Matrix identities.* Check that the following identities regarding matrix inverses hold. You can assume that X, Y, Z are matrices in appropriate sizes, and a, b are vectors in appropriate sizes. You can also assume that the appearing inverses exist.

a)

$$Z(I + Z)^{-1} = I - (I + Z)^{-1}$$

b)

$$(I + XY)^{-1} = I - X(I + YX)^{-1}Y$$

c)

$$Y(I + XY)^{-1} = (I + YX)^{-1}Y$$

d)

$$(I + XZ^{-1}Y)^{-1} = I - X(Z + YX)^{-1}Y$$

e)

$$(X + ab^T)^{-1} = X^{-1} - \frac{1}{1 + b^T X^{-1} a} X^{-1} ab^T X^{-1}$$

4) *VMLS Exercises.*

- a) **11.6** *Inverse of a block upper triangular matrix.*
- b) **11.12** *Combinations of invertible matrices.*
- c) **11.14** *Middle inverse.*
- d) **11.15** *Invertibility of population dynamics matrix.*