ASE2910 Applied Linear Algebra / AUS2910 Fundamental Math for AI
Homework #4

1) *Lower triangular matrices.* Let A be a lower triangular $n \times n$ matrix. Verify the following properties.

   a) If B is a lower triangular $n \times n$ matrix, then the product AB is lower triangular.

   b) The matrix A^k is lower triangular for all positive integers k.

   c) If A is invertible, then A^k is lower triangular for all integers k (positive or negative).

2) *QR factorization of matrix with orthonormal columns.* Suppose the columns of a matrix A are orthonormal, and we (attempt) to compute its QR factorization $A = QR$. Which of the following must be true?

   a) The QR factorization will fail.

   b) $R = I$

   c) $R = A$

   d) $Q = I$

   e) $Q = A$

3) *Matrix identities.* Check that the following identities regarding matrix inverses hold. You can assume that X,Y,Z are matrices in appropriate sizes, and a, b are vectors in appropriate sizes. You can also assume that the appearing inverses exist.

   a)
   $$Z(I+Z)^{-1} = I - (I+Z)^{-1}$$

   b)
   $$(I+XY)^{-1} = I - X(I+YX)^{-1}Y$$

   c)
   $$Y(I+XY)^{-1} = (I+YX)^{-1}Y$$

   d)
   $$(I+XZ^{-1}Y)^{-1} = I - X(Z+YX)^{-1}Y$$

   e)
   $$(X+ab^T)^{-1} = X^{-1} - \frac{1}{1+b^TX^{-1}a}X^{-1}ab^TX^{-1}$$

4) *VMLS Exercises.*

   a) **11.6** *Inverse of a block upper triangular matrix.*

   b) **11.12** *Combinations of invertible matrices.*

   c) **11.14** *Middle inverse.*

   d) **11.15** *Invertibility of population dynamics matrix.*