

**ASE3093 Automatic Control: Final Exam (1 big problem, 90 minutes)**

Before the exam begins, write the following ‘Student Honor Code’ at the top of your answer sheet and sign it: **“By signing this pledge, I promise to adhere to exam requirements and maintain the highest level of ethical principles during the exam period.”**

시험 시작 전, 다음의 '학생 명예선서(Honor Code)'를 답지 맨 위에 적고 서명하시오:  
**“나는 정직하게 시험에 응할 것을 서약합니다.”**

1) *Silver Star’s Life on Mars (40pts).*

Following its initial success navigating the Martian terrain, *Silver Star*, the rover developed by the ACSL team at Inha University, has entered a sustained mission phase. The rover is expected to perform autonomous traverses, scientific data collection, and occasionally respond to teleoperated commands sent from Earth. Due to prolonged operation under harsh Martian conditions, the engineering team must continually analyze, tune, and reinforce the stability and performance of its control systems.



The rover’s longitudinal motion, influenced by the wheel actuators and terrain resistance, is modeled by:

$$\begin{aligned} m\dot{v}(t) + bv(t) &= u(t), \\ \dot{x}(t) &= v(t) + d(t), \end{aligned}$$

where  $x(t)$  is the rover’s position,  $v(t)$  is its velocity,  $u(t)$  is the control force generated by its motor, and  $d(t)$  represents the unknown terrain disturbance (e.g., caused by slope or dust drag). The physical parameters are  $m = 1$  (mass) and  $b = 4$  (damping).

**Initial Surface Deployment Stage: PI Control**

To overcome terrain disturbances and achieve accurate position tracking, the rover’s onboard computer applies a PI controller:

$$\begin{aligned} u(t) &= K_p(x_d(t) - x(t)) + K_i\epsilon(t), \\ \dot{\epsilon}(t) &= x_d(t) - x(t). \end{aligned}$$

where  $x_d(t)$  is the desired trajectory, and the control gains are positive numbers, i.e.,  $K_p > 0$  and  $K_i > 0$ .

- During early mission trials, the control team adjusted  $K_i$  while keeping  $K_p = 6$  fixed. Sketch the root locus with respect to  $K_i$ . Describe the general pole movement and its implications for stability and response. (4pts)
- Conversely, with  $K_i = 25$  fixed, sketch the root locus with respect to  $K_p$ . Comment on whether increasing  $K_p$  improves or degrades system performance. (4pts)

- c) For fast and well-damped position control, the team targets closed-loop poles at  $s = -1 \pm j$ . Calculate  $(K_p, K_i)$  values that place the closed-loop system's poles at these locations. (4pts)
- d) For long-term autonomy, the team seeks a robust set of gains. Derive the region in the  $(K_p, K_i)$  plane where the closed-loop system remains stable. (4pts)

Based on the preceding analyses, the design team finalized and implemented the PI controller parameters as follows:

$$(K_p, K_i) = (5, 5).$$

### Aging Silver Star: Model Uncertainty

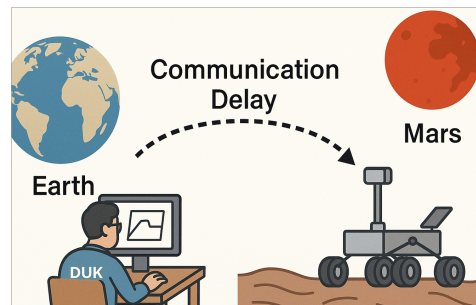
As the rover's operation on Mars continues, actuator wear and thermal fluctuations begin degrading the motor's effective output. This is modeled by a scale factor  $\eta > 0$  as follows:

$$\begin{aligned} m\dot{v}(t) + bv(t) &= \eta u(t), \\ \dot{x}(t) &= v(t) + d(t) \end{aligned}$$

- e) Determine the range of  $\eta$  for which the closed-loop system remains stable. Use root locus or frequency-domain arguments to justify your answer. (4pts)
- f) The design team observed a slight degradation in stability performance and decided to adjust the PI controller gains by introducing a scaling factor  $\alpha > 0$ , resulting in controller parameters  $(\alpha K_p, \alpha K_i)$ . Determine the value of  $\alpha$  that maximizes the system's phase margin. Assume that  $\eta = 1$ . (6pts)

### Teleoperation Phase: Delay-Induced Instability

Certain scientific tasks—such as high-precision sampling near hazardous obstacles—require teleoperation rather than full autonomy. During these operations, computationally intensive processes like image processing or target recognition, as well as the PI control computation are handled by the mission control team on Earth.



However, due to the vast distance between Earth and Mars, all control commands issued from Earth experience a one-way communication delay of  $\tau$  seconds before reaching the rover.

$$u(t) = u_c(t - \tau)$$

where  $u_c(t)$  is the control signal calculated on Earth (as in earlier parts). The rover dynamics are (with perfect motor condition  $\eta = 1$ ) still:

$$\begin{aligned} m\dot{v}(t) + bv(t) &= u(t) \\ \dot{x}(t) &= v(t) + d(t), \end{aligned}$$

and the controller we use is the one with the integral action:

$$u_c(t) = K_p (x_d(t) - x(t)) + K_i \epsilon(t),$$

$$\dot{\epsilon}(t) = x_d(t) - x(t).$$

Note that the control command is now represented by  $u_c(t)$  rather than the immediate input  $u(t)$ , due to the communication delay. This delay introduces a phase lag into the control loop, which may degrade system stability or even lead to oscillatory or unstable behavior. In the following, you are tasked with analyzing the impact of this communication delay on the position control system of the Silver Star rover.

- g) Plot the frequency response (Bode plot) of the delay element (transfer function from  $u_c(t)$  to  $u(t)$ ). (4pts)
- h) Provide an analytical justification that there exists a critical time delay  $\tau_c$  beyond which the closed-loop system becomes unstable. Support your argument using frequency domain analysis and appropriate stability criteria. (4pts)
- i) Based on data collected during initial on-site operations, the ACSL team successfully identified the frequency response of the open-loop system, which includes both the Silver Star's dynamics and the PI controller. Using this information, determine the critical time delay  $\tau_c$  beyond which the closed-loop system becomes unstable. (6pts)

