

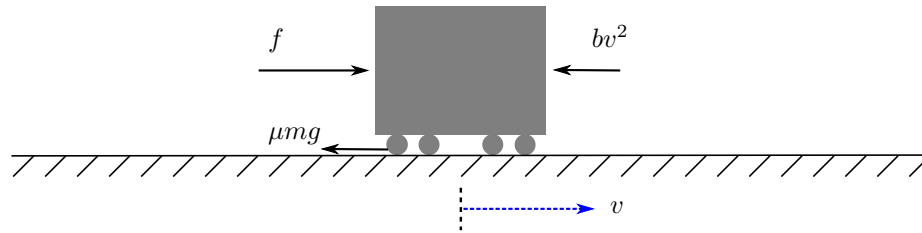
ASE3093 Automatic Control: Homework #1

- 1) *Linearization of cruise control system.* In this problem, we model a cruise control system for a ground vehicle.

First, consider the vehicle dynamics with aerodynamic drag and ground friction. Let:

- m : mass of the vehicle,
- v : velocity of the vehicle,
- bv^2 : aerodynamic drag,
- μmg : constant ground friction,
- f : thrust force generated by the engine.

The cruise control system calculates the thrust force f so that the vehicle moves at a desired speed v_r (i.e., $v \approx v_r$), and the control system commands the engine to generate the required thrust force f .



- a) Write the equation of motion for the above system. It should be a nonlinear differential equation in v , using the parameters and variables defined above.
- b) Since nonlinear equations are difficult to analyze and design controllers for, we linearize the equation around an equilibrium point defined by the desired speed v_r . Assume the car is cruising steadily at v_r . What is the corresponding propulsion force f_r ?
- c) Define new variables:

$$x = v - v_r$$

$$u = f - f_r$$

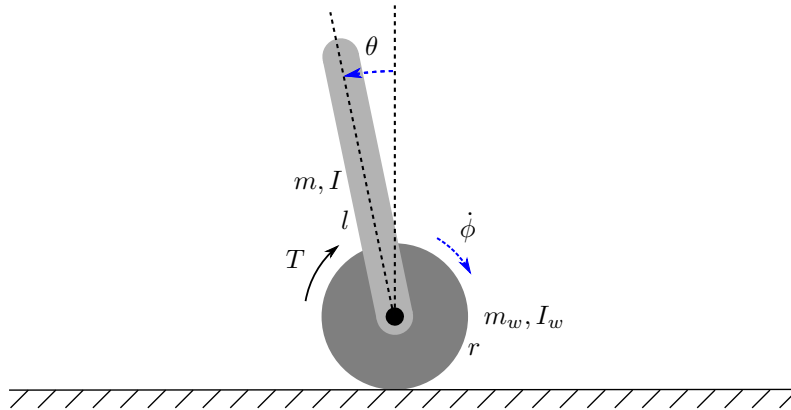
Here, x is the deviation from equilibrium speed, and u is the deviation from equilibrium propulsion force. Assuming x and u are small, linearize the nonlinear equation to express it as a linear differential equation in terms of x and u .

- 2) *Segway (10 points)*. We analyze the dynamics of a Segway consisting of a platform and a wheel. The platform, along with the rider, is modeled as a uniform stick. It is connected to the wheel by a motor capable of applying a control torque T . This motor rotates the wheel and helps balance the platform.

When torque T is applied to the motor:

- The wheel experiences a clockwise torque $+T$,
- The platform experiences an equal and opposite counterclockwise torque $-T$.

Assume sufficient amount of static friction exists between the wheel and the ground so it rolls without slipping.



- m : mass of the platform
- $2l$: length of the platform (center of mass at the midpoint)
- I : moment of inertia of the platform
- θ : tilt angle from vertical
- m_w, r : mass and radius of the wheel
- I_w : moment of inertia of the wheel
- $\dot{\phi}$: angular velocity of the wheel
- g : gravitational acceleration

The dynamics are expressed as:

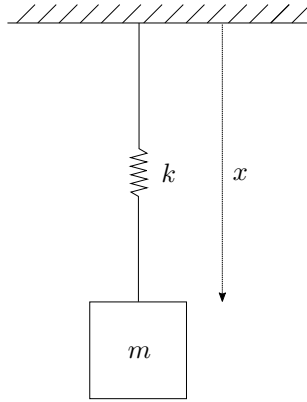
$$\begin{bmatrix} I + ml^2 & -mrl \cos \theta \\ -mrl \cos \theta & I_w + (m + m_w)r^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} T + mgl \sin \theta \\ T - mrl \dot{\theta}^2 \sin \theta \end{bmatrix}$$

To accelerate the Segway, it suffices to control the tilt angle θ to a constant desired angle θ_{cmd} . The controller computes control torque T using θ and $\dot{\theta}$ to maintain $\theta \approx \theta_{\text{cmd}}$.

- a) Assume the controller keeps $\theta \approx \theta_{\text{cmd}}$, where $0 < \theta_{\text{cmd}} < \pi/4$. In which direction and how fast does the Segway accelerate? Express your answer using the given parameters.

- 3) *Nonlinear spring.* A mass m hangs from a spring attached to the ceiling. When no mass is attached, the spring's length is x_0 . When the spring stretches to length x , the restoring force is:

$$F_s = k(x - x_0)^3 \quad (\text{for } x > x_0)$$



Initially, the mass is gently attached and rests in equilibrium. Then, a fly bumps into the mass vertically (in the $-x$ direction). What is the period of oscillation of the mass? Express your answer in terms of m , g , and k only.

- 4) *Solutions of homogeneous linear ordinary differential equations.* Find $x(t)$ for each of the following cases, given the initial conditions $x(0) = 0$ and $\dot{x}(0) = v_0$.

- a) $x(t)$ satisfies the equation:

$$m\ddot{x} + b\dot{x} = 0$$

- b) $x(t)$ satisfies the equation:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2x = 0$$