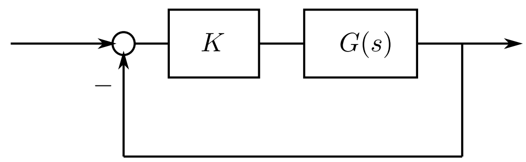
ASE3093 Automatic Control: Homework #4

1) *Root locus.* Draw the root locus for the following systems, appropriately indicating departure/arrival angles, asymptotes, and the center of the asymptotes. Assume the controller K is a positive constant ($K > 0$). Analyze the stability of the closed-loop system.



a) $$G(s) = \frac{1}{s(s+1)}$$

b) $$G(s) = \frac{s+2}{s(s+1)}$$

c) $$G(s) = \frac{1}{s(s+1)(s+2)}$$

d) $$G(s) = \frac{s+2}{s^2(s+20)}$$

e) $$G(s) = \frac{s+2}{s^2(s+3)}$$

f) $$G(s) = \frac{1}{s(s-1)}$$

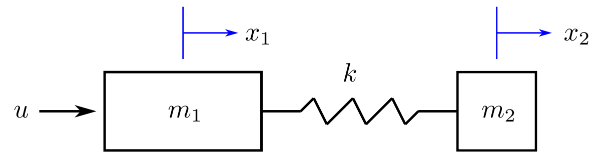
g) $$G(s) = \frac{s+2}{s(s-1)}$$

h) $$G(s) = \frac{s-1}{(s-2)(s+10)}$$

i) $$G(s) = \frac{s-1}{(s-2)(s-3)(s+10)}$$

j) $$G(s) = \frac{(s+0.5)(s+1.5)}{s(s^2+2s+2)(s+5)(s+15)}$$

2) *Collocated vs. noncollocated systems.* Consider a system in which two masses m_1 and m_2 are connected by a spring with constant k, and a control force u is applied to m_1. Given $m_1 = 10$, $m_2 = 1$, and $k = 100$, answer to the following questions:

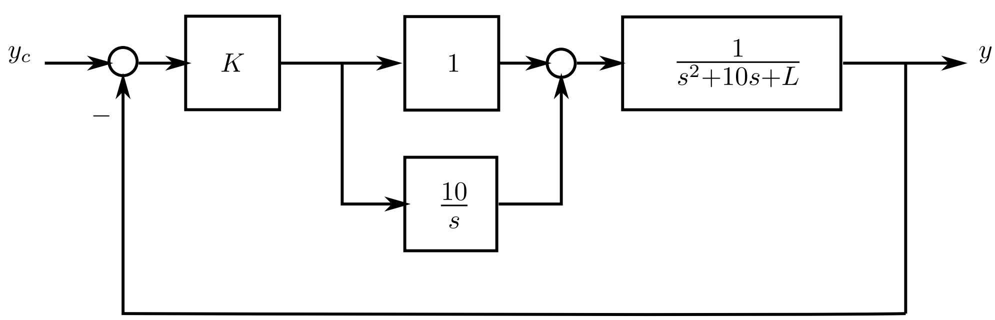


a) Assume that a position sensor is attached to m_1, and the measured signal x_1 is used for computing the control input u (this configuration is called *collocated*). Derive the transfer function $G_c(s) = x_1(s)/u(s)$.

b) Assume that a position sensor is attached to m_2, and the measured signal x_2 is used for computing the control input u (this configuration is called *noncollocated*). Derive the transfer function $G_{nc}(s) = x_2(s)/u(s)$.

c) Show that the collocated system $G_c(s)$ can be stabilized using a PD controller. In contrast, show that no PD controller can stabilize the noncollocated system $G_{nc}(s)$.

3) *Two-parameter system.* In the feedback loop shown below, $K > 0$ denotes the controller gain and $L > 0$ is a parameter of the plant. Investigate how the location of the closed-loop poles changes as K and L vary.

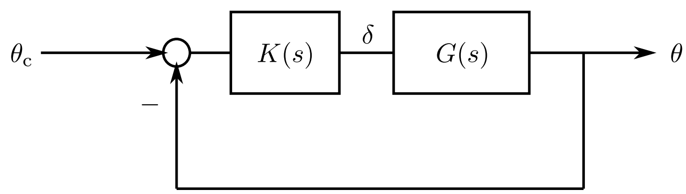


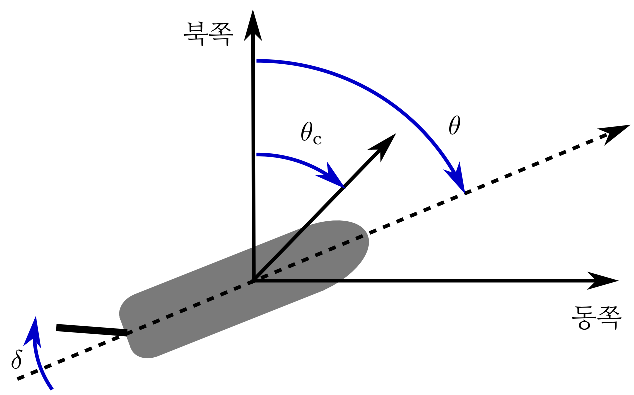
a) With the plant parameter fixed at $L = 50$, sketch the root locus of the closed-loop poles in the complex plane as the controller gain K varies over $0 < K < \infty$.

b) With the controller gain fixed at $K = 50$, sketch the root locus of the closed-loop poles in the complex plane as the plant parameter L varies over $0 < L < \infty$.

4) *Course correction autopilot.* The following figure represents a course autopilot for a vessel system. The output variable θ and control input δ represent the *heading angle* and *fin deflection*, respectively. The objective is to design a controller $K(s)$ that satisfies the following two performance requirements:

- **Req.#1.** The autopilot system must track step and ramp reference commands θ_c with zero steady-state error.
- **Req.#2.** The closed-loop damping ratio should be close to $1/\sqrt{2}$.





The heading angle dynamics with respect to the fin deflection are described by:

$$G(s) = \frac{s+1}{s^2(s-0.1)},$$

and the following controller structures are considered for $K(s)$:

- P control: $K(s) = K$,
- PI control: $K(s) = K\left(1 + \frac{3}{s}\right)$,
- PD control: $K(s) = K(2s+1)$.

a) Among the candidate controllers, select the one that satisfies **Req.#1**, and justify your answer.

b) For the selected controller in part (a), determine a gain K such that **Req.#2** is satisfied. You may use computational tools as necessary.