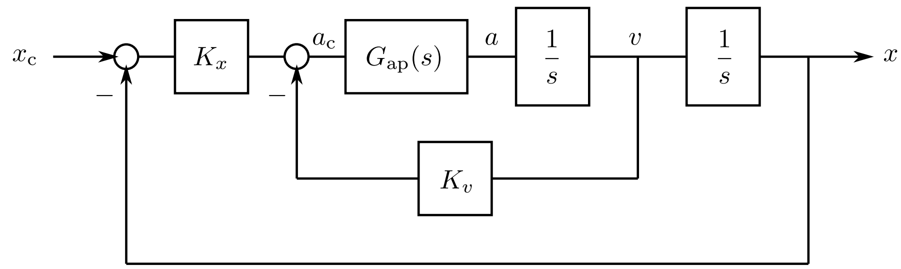
ASE3093 Automatic Control: Homework #6

1) *Runway approach problem.* Your job is to design a controller that computes the acceleration command, a_c, from the lateral deviation, x, and the lateral velocity, v, that is, to choose K_v and K_x, and to check the robustness of your design. The block diagram describing the dynamics of the considered system is shown below.



Your controller computes the acceleration command, a_c, which is sent to the autopilot that somehow generates the actual acceleration response, a, through $G_{ap}(s) = a(s)/a_c(s)$. For now, assume that the autopilot is ideal, $G_{ap}(s) = 1$.

a) Find K_v and K_x that place the closed loop pole at $s = -1 \pm j$, so that the closed loop bandwidth is 2 and the closed loop damping is $1/\sqrt{2}$.

Now fix K_v and K_x by the ones you found in a), and we assume that K_x is disturbed by a scale factor $\xi > 0$, so the new position gain can be ξK_x while the velocity gain stays the same as K_v.

b) For what range of ξ, is the closed loop system stable? You may use computational tools to check the stability margin of your design.

A more realistic autopilot can be modelled by a third order system as

$$G_{ap}(s) = \frac{a(s)}{a_c(s)} = \frac{p\omega^2}{(s+p)(s^2 + 2\zeta\omega s + \omega^2)}$$

where we let $\omega = 4$, $\zeta = 0.7$, and $p = 6$.

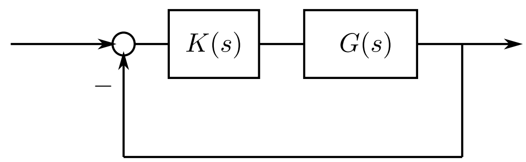
c) Under presence of this autopilot model, for what range of ξ, is the closed loop system stable? You will need to use the computational tools to check the stability margin of your design.

2) *Nyquist stability criterion.* Consider the following double integrator system

$$G(s) = \frac{1}{s^2}$$

with a PID control

$$K(s) = 2(1 + \frac{1}{s} + s)$$



a) What is the magnitude and the phase of $K(s)G(s)$ at $\omega = 1$ ($s = j$)?

b) Carefully draw the Bode diagrams. You may check your results with the `bode()` function.

c) What is the gain margin and the phase margin of your control system? You may check your results with the `margin()` function. Your answer may look unfamiliar. What does that mean?

d) Carefully draw the Nyquist diagram with extra caution at around $\omega \to 0$ (the infinite radius parts in your diagram). Unfortunately, using the `nyquist()` function won't help in this specific problem.

e) Check the closed loop stability from the diagram you obtained in d). What is N (the number of the clockwise encirclements around -1), and Z (the number of unstable poles in your closed loop system)? Is the closed loop system stable?

f) Check the stability result that you've obtained in e) by using computational tools. You may use the `rlocus()` function or whatever (*e.g.*, `pole()`, `eig()`, *et cetera*), to check your closed loop pole locations.

g) Your answer in f) should correspond to what you've got in d)-e). If so, you are good. If not, go back and think about it, or discuss with your friends.