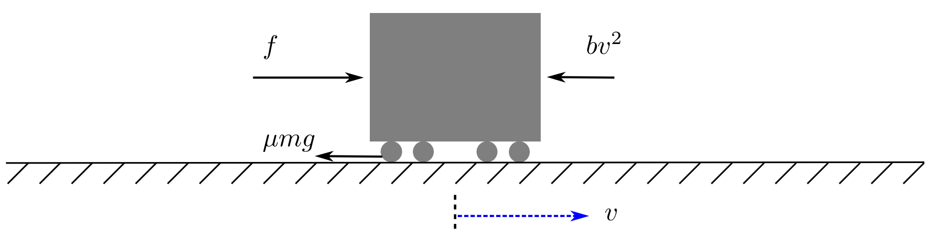
ASE3093 Automatic Control / AUS3204 Applied Control Engineering
Homework #1

1) *Linearization of cruise control system.* In this problem, we model a cruise control system for a ground vehicle.

   First, consider the vehicle dynamics with aerodynamic drag and ground friction. Let:

   - m: mass of the vehicle,
   - v: velocity of the vehicle,
   - bv^2: aerodynamic drag,
   - μmg: constant ground friction,
   - f: thrust force generated by the engine.

   The cruise control system calculates the thrust force f so that the vehicle moves at a desired speed v_r (*i.e.*, $v \approx v_r$), and the control system commands the engine to generate the required thrust force f.



   a) Write the equation of motion for the above system. It should be a nonlinear differential equation in v, using the parameters and variables defined above.

   b) Since nonlinear equations are difficult to analyze and design controllers for, we linearize the equation around an equilibrium point defined by the desired speed v_r. Assume the car is cruising steadily at v_r. What is the corresponding propulsion force f_r?

   c) Define new variables:

$$x = v - v_r$$
$$u = f - f_r$$

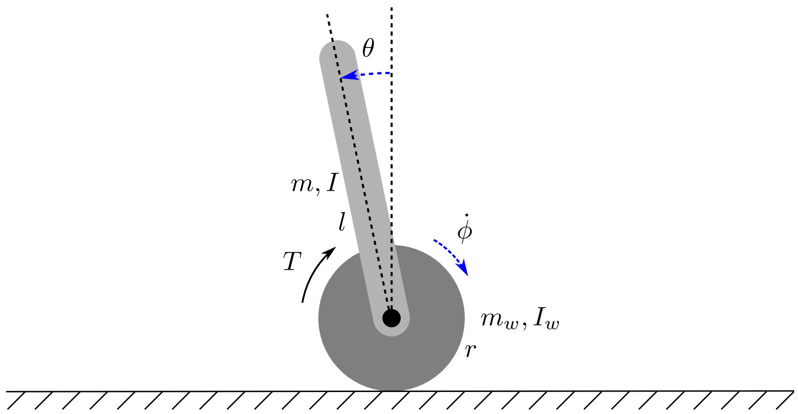
   Here, x is the deviation from equilibrium speed, and u is the deviation from equilibrium propulsion force. Assuming x and u are small, linearize the nonlinear equation to express it as a linear differential equation in terms of x and u.

2) *Segway (10 points).* We analyze the dynamics of a Segway consisting of a platform and a wheel. The platform, along with the rider, is modeled as a uniform stick. It is connected to the wheel by a motor capable of applying a control torque T. This motor rotates the wheel and helps balance the platform.

When torque T is applied to the motor:

- The wheel experiences a clockwise torque $+T$,
- The platform experiences an equal and opposite counterclockwise torque $-T$.

Assume sufficient amount of static friction exists between the wheel and the ground so it rolls without slipping.



- m: mass of the platform
- $2l$: length of the platform (center of mass at the midpoint)
- I: moment of inertia of the platform
- θ: tilt angle from vertical
- m_w, r: mass and radius of the wheel
- I_w: moment of inertia of the wheel
- $\dot{\phi}$: angular velocity of the wheel
- g: gravitational acceleration

The dynamics are expressed as:

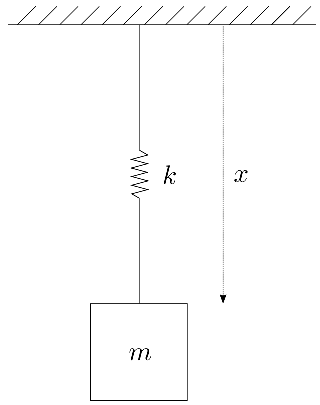
$$\begin{bmatrix} I + ml^2 & -mrl\cos\theta \\ -mrl\cos\theta & I_w + (m + m_w)r^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} T + mgl\sin\theta \\ T - mrl\dot{\theta}^2\sin\theta \end{bmatrix}$$

To accelerate the Segway, it suffices to control the tilt angle θ to a constant desired angle θ_{cmd}. The controller computes control torque T using θ and $\dot{\theta}$ to maintain $\theta \approx \theta_{\text{cmd}}$.

a) Assume the controller keeps $\theta \approx \theta_{\text{cmd}}$, where $0 < \theta_{\text{cmd}} < \pi/4$. In which direction and how fast does the Segway accelerate? Express your answer using the given parameters.

3) *Nonlinear spring.* A mass m hangs from a spring attached to the ceiling. When no mass is attached, the spring's length is x_0. When the spring stretches to length x, the restoring force is:

$$F_s = k(x - x_0)^3 \quad (\text{for } x > x_0)$$



Initially, the mass is gently attached and rests in equilibrium. Then, a fly bumps into the mass vertically (in the $-x$ direction). What is the period of oscillation of the mass? Express your answer in terms of m, g, and k only.

4) *Solutions of homogeneous linear ordinary differential equations.* Find $x(t)$ for each of the following cases, given the initial conditions $x(0) = 0$ and $\dot{x}(0) = v_0$.

a) $x(t)$ satisfies the equation:

$$m\ddot{x} + b\dot{x} = 0$$

b) $x(t)$ satisfies the equation:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = 0$$