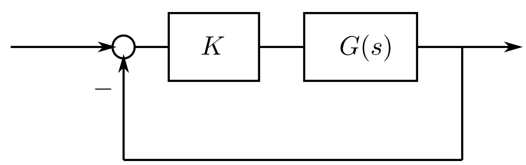
# ASE3093 Automatic Control / AUS3204 Applied Control Engineering
# Homework #4

1) *Root locus.* Draw the root locus for the following systems, appropriately indicating departure/arrival angles, asymptotes, and the center of the asymptotes. Assume the controller $K$ is a positive constant ($K > 0$). Analyze the stability of the closed-loop system.




a) $G(s) = \dfrac{1}{s(s+1)}$

b) $G(s) = \dfrac{s+2}{s(s+1)}$

c) $G(s) = \dfrac{1}{s(s+1)(s+2)}$

d) $G(s) = \dfrac{s+2}{s^2(s+20)}$

e) $G(s) = \dfrac{s+2}{s^2(s+3)}$

f) $G(s) = \dfrac{1}{s(s-1)}$

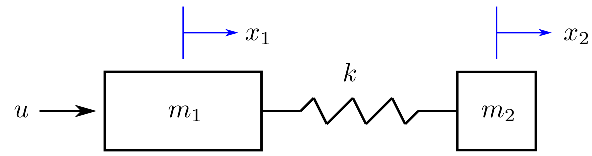
g) $G(s) = \dfrac{s+2}{s(s-1)}$

h) $G(s) = \dfrac{s-1}{(s-2)(s+10)}$

i) $G(s) = \dfrac{s-1}{(s-2)(s-3)(s+10)}$

j) $G(s) = \dfrac{(s+0.5)(s+1.5)}{s(s^2+2s+2)(s+5)(s+15)}$

2) *Collocated vs. noncollocated systems.* Consider a system in which two masses $m_1$ and $m_2$ are connected by a spring with constant $k$, and a control force $u$ is applied to $m_1$. Given $m_1 = 10$, $m_2 = 1$, and $k = 100$, answer to the following questions:

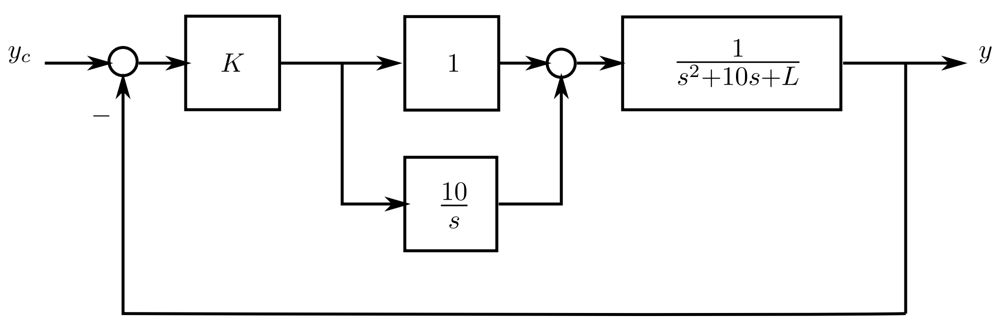



a) Assume that a position sensor is attached to $m_1$, and the measured signal $x_1$ is used for computing the control input $u$ (this configuration is called *collocated*). Derive the transfer function $G_c(s) = x_1(s)/u(s)$.

b) Assume that a position sensor is attached to $m_2$, and the measured signal $x_2$ is used for computing the control input $u$ (this configuration is called *noncollocated*). Derive the transfer function $G_{nc}(s) = x_2(s)/u(s)$.

c) Show that the collocated system $G_c(s)$ can be stabilized using a PD controller. In contrast, show that no PD controller can stabilize the noncollocated system $G_{nc}(s)$.

3) *Two-parameter system.* In the feedback loop shown below, $K > 0$ denotes the controller gain and $L > 0$ is a parameter of the plant. Investigate how the location of the closed-loop poles changes as $K$ and $L$ vary.

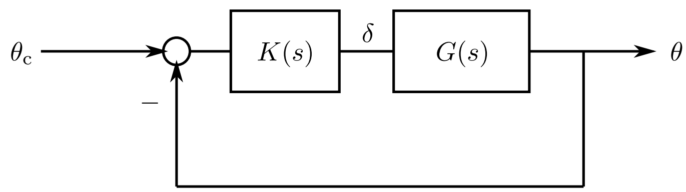


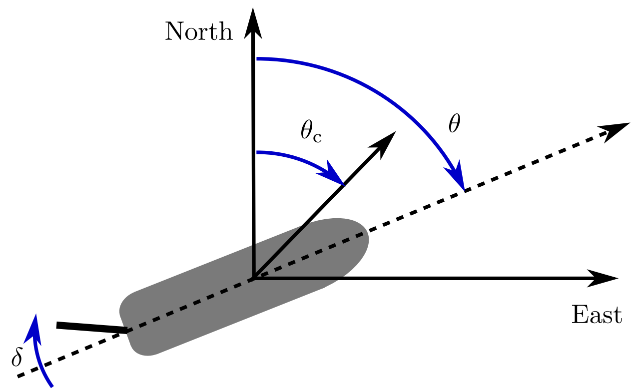

a) With the plant parameter fixed at $L = 50$, sketch the root locus of the closed-loop poles in the complex plane as the controller gain $K$ varies over $0 < K < \infty$.

b) With the controller gain fixed at $K = 50$, sketch the root locus of the closed-loop poles in the complex plane as the plant parameter $L$ varies over $0 < L < \infty$.

4) *Course correction autopilot.* The following figure represents a course autopilot for a vessel system. The output variable $\theta$ and control input $\delta$ represent the *heading angle* and *fin deflection*, respectively. The objective is to design a controller $K(s)$ that satisfies the following two performance requirements:

- **Req.#1.** The autopilot system must track step and ramp reference commands $\theta_c$ with zero steady-state error.
- **Req.#2.** The closed-loop damping ratio should be close to $1/\sqrt{2}$.




The heading angle dynamics with respect to the fin deflection are described by:

$$G(s) = \frac{s+1}{s^2(s-0.1)},$$

and the following controller structures are considered for $K(s)$:

- P control: $K(s) = K,$
- PI control: $K(s) = K\left(1 + \frac{3}{s}\right),$
- PD control: $K(s) = K\,(2s+1).$

a) Among the candidate controllers, select the one that satisfies **Req.#1**, and justify your answer.

b) For the selected controller in part (a), determine a gain $K$ such that **Req.#2** is satisfied. You may use computational tools as necessary.