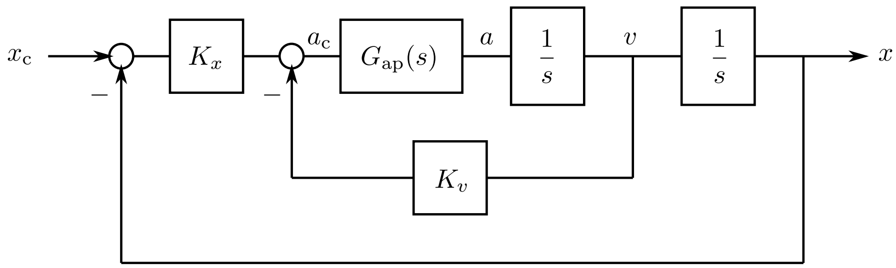
ASE3093 Automatic Control / AUS3204 Applied Control Engineering Homework #6

1) *Runway approach problem.* The following represents the lateral path control system of an aircraft approaching a runway. In this problem, we want to design a controller to calculate the lateral maneuver acceleration command a_c from the position error $x_{\mathrm{c}} - x$ and the velocity error v (by selecting K_x and K_v), and verify the robustness of the designed controller. The dynamics of the system can be represented by the block diagram below.



The designed controller calculates the maneuver acceleration command a_{c}, and this maneuver acceleration command a_{c} is transmitted to the autopilot $G_{\mathrm{ap}}(s) = a(s)/a_{\mathrm{c}}(s)$ to generate the actual acceleration a.

First, assume that the autopilot is ideal, so $G_{\mathrm{ap}}(s) = 1$. That is, assume the autopilot instantaneously and accurately generates the maneuver acceleration command calculated by the controller.

a) Determine the values of K_v and K_x such that the closed-loop poles are located at $s = -1 \pm j$, resulting in a closed-loop bandwidth and damping of $\sqrt{2}$ and $1/\sqrt{2}$, respectively.

Now, assume that a scale factor error exists in the autopilot such that $G_{\mathrm{ap}}(s) = \xi$ instead of $G_{\mathrm{ap}}(s) = 1$. Here, ξ is a positive real number.

b) When the controller designed in (a) is applied to the system considering $G_{\mathrm{ap}}(s) = \xi$, find the range of ξ that guarantees the stability of the closed-loop system. Use a computer if necessary.

A more realistic autopilot can be modeled as the following 3rd-order dynamic system:

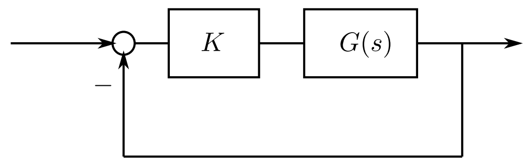
$$G_{\mathrm{ap}}(s) = \frac{a(s)}{a_{\mathrm{c}}(s)} = \frac{\xi p \omega^2}{(s+p)(s^2 + 2\zeta\omega s + \omega^2)}$$

In the above system, assume $\omega = 4$, $\zeta = 0.7$, and $p = 6$.

c) When the controller designed in (a) is applied to the system considering the 3rd-order autopilot above, find the range of ξ that guarantees the stability of the closed-loop system. Use a computer if necessary.

2) *Stability margin of statically unstable systems.* Consider an open-loop unstable system $G(s)$, and a proportional control strategy using a constant gain K:

$$G(s) = \frac{100(s+1)}{s(s-1)(s^2+10s+50)}$$

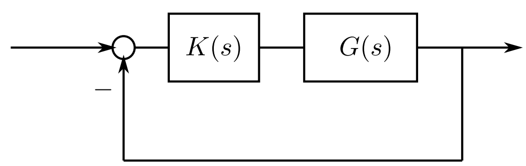


Determine the range of K that stabilizes the system. Use Bode plot and Root Locus analysis to evaluate the stability margin comprehensively. Computer-based tools may be used to aid in the analysis.

3) *Nyquist stability criterion.* Consider the following double integrator system utilizing PID control. Answer the questions below.

$$G(s) = \frac{1}{s^2}$$

$$K(s) = 2(1 + \frac{1}{s} + s)$$



a) When $\omega = 1$ ($s = j$), find the magnitude and phase of $K(s)G(s)$.

b) Draw the Bode plot by hand, and compare it with the result using the MATLAB `bode()` function.

c) Find the gain margin and phase margin of this control system, and compare them with the result using the MATLAB `margin()` function. Check how the results differ and describe why such results were obtained.

d) Draw the Nyquist plot, paying attention to the point where $\omega \to 0$ (the point with an infinite radius in the diagram). In this problem, the MATLAB `nyquist()` function will not be of much help.

e) Check the closed-loop stability from the Nyquist plot obtained in d), and find the values of N (the number of clockwise encirclements of the -1 point) and Z (the number of unstable poles in the closed-loop system). Finally, confirm the stability of the closed-loop system.

f) Compare the stability confirmation result obtained in e) with the result utilizing MATLAB. Use the `rlocus()` function or (*e.g.*, `pole(), eig()`, *et cetera*) to check the pole locations of the closed-loop system.

g) The result of f) must be identical to the results of d)-e). If not, discuss with your friends and solve the above problem again.