ASE6029 Linear optimal control: Homework #1

1) *Diagonalization.* Show that a matrix with distinct eigenvalues is diagonalizable.

2) *Symmetric matrices.*

   a) Show that a symmetric matrix has real eigenvalues.

   b) Show that a symmetric matrix with distinct eigenvalues is orthogonally diagonalizable.

   c) Show that a symmetric matrix is orthogonally diagonalizable.

   d) Say the eigenvalues of $A \in \mathbb{S}^n$ are ordered as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. Show that

   $$\lambda_n \|x\|^2 \leq x^T A x \leq \lambda_1 \|x\|^2$$

   and explain when the inequalities are tight.

3) *Simultaneous diagonalizability.* Two matrices A and B are said to be simultaneously diagonalizable if there exists an invertible matrix T such that both $T^{-1}AT$ and $T^{-1}BT$ are diagonal. Now supppose that A and B are simultaneously diagonalizable.

   a) Show that they commute:

   $$AB = BA$$

   b) Show that the product rule for matrix exponentials holds in this case..

   $$e^{A+B} = e^A e^B$$