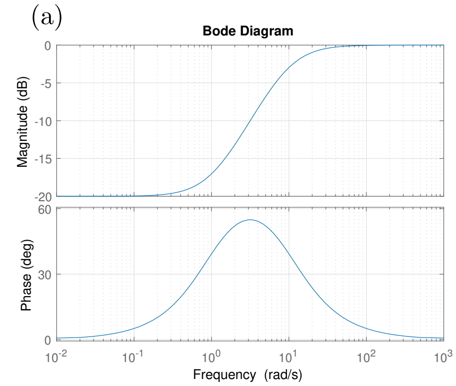
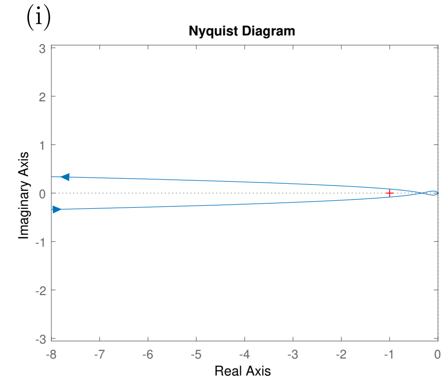
EE363 Automatic Control: Final Exam (4 problems, 75 minutes)

1) *Matching diagrams (10 points).* You should now be very familiar with the diagrams below. Draw a line to the matching plot that came from the same transfer function. No explanation required.

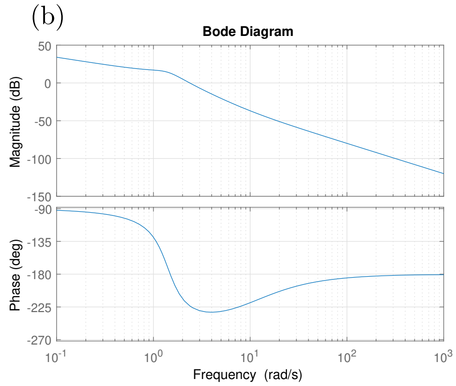
(a)



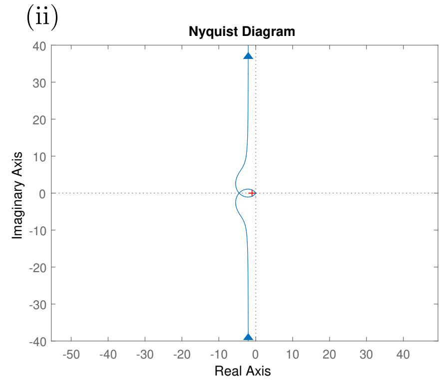
(i)



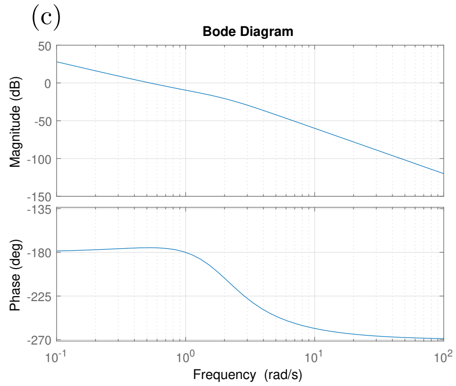
(b)



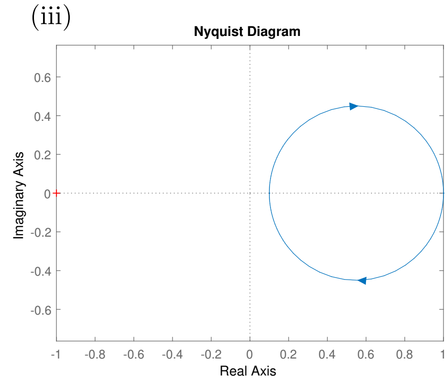
(ii)



(c)



(iii)



Solution: By inspecting the phase at $\omega \to 0^+$, we have that (a)$\to$(iii), (b)$\to$(ii), and (c)$\to$(i).

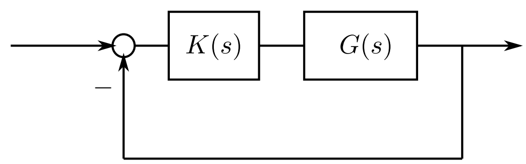
2) *PD control (10 points).* Conider the following third order plant.

$$G(s) = \frac{1}{s^2(s+1)}$$

With a simple PD controller of the following form with $K_d > 0$,

$$K(s) = K_d s + K_p$$

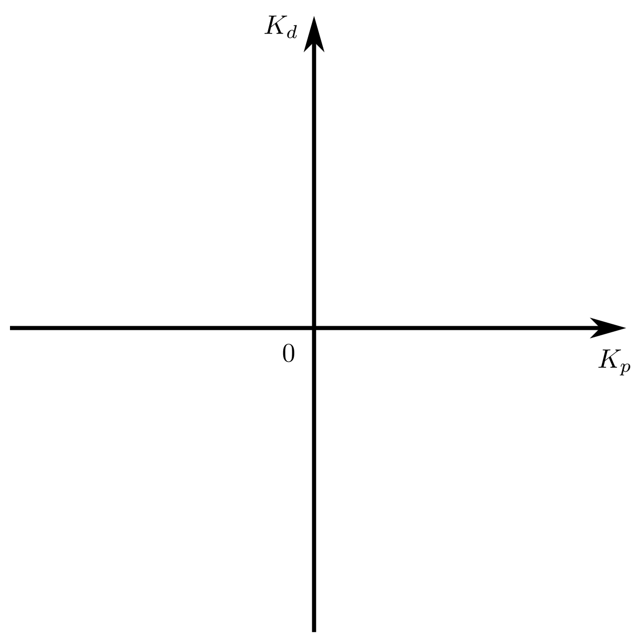
your job is to identify the set of all stabilizing controllers, *i.e.*, to find the set of all K_p and K_d that stabilizes the following closed loop system.



Note that you don't need to find good controllers; you are ok as long as your controllers stabilize the closed loop system.

a) Parameterize all stabilizing controllers, *i.e.*, find the conditions under which the closed loop system is stable. Your answer should be in terms of K_p and K_d.

b) On the following 2D graph, explicitly shadow the region occupied by the stabilizing controllers.

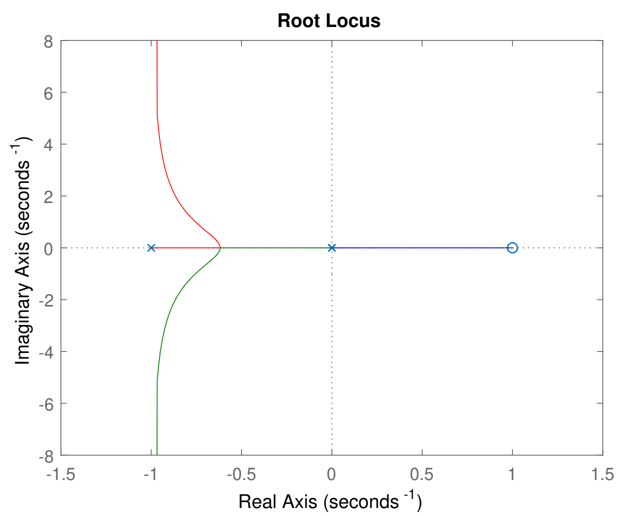


Solution:

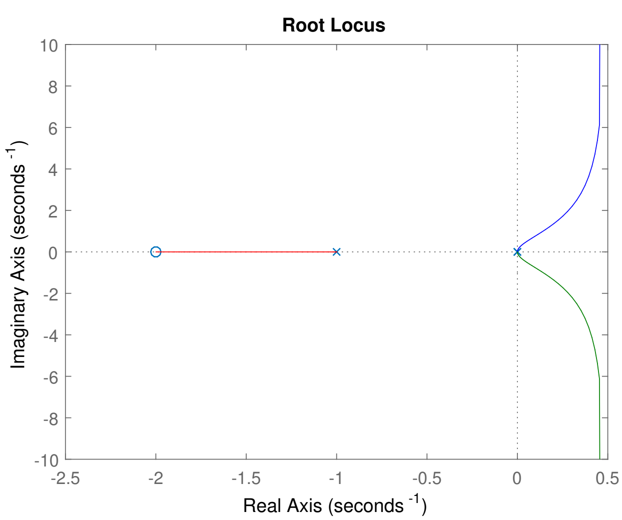
a) We will examine the root loci for different set of K_p's, noting that K_d is positive. Since the open loop transfer function has three poles and one zero, we have two asymptotes heading towards ± 90 deg, with the center located at

$$\frac{-1 + K_p/K_d}{2}$$

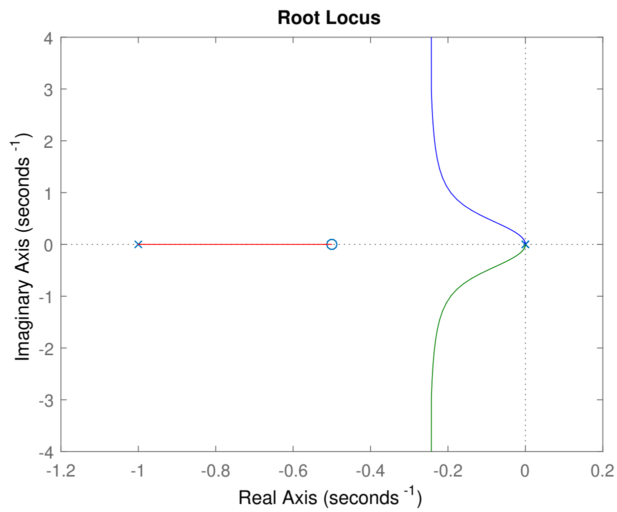
First we consider $K_p < 0$. Then the departure angles of the two poles at the origin are 0 deg and 180 deg, which implies that $K_d < 0$ fails to stabilize the closed loop system. For example, the root locus with $K_p = -1$ looks like this.



Now we consider $K_p \geq K_d$. Then the departure angles of the two poles at the origin are ± 90 deg, and center of the asymptotes lies on the non-negative real axis, which implies that $K_p \geq K_d$ also fails to stabilize the closed loop system. For example, the root locus with $K_p = 2K_d$ looks like the following.

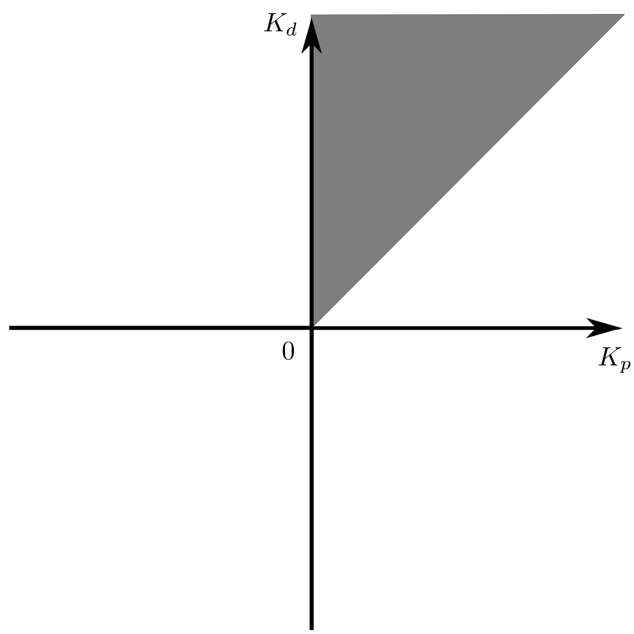


Finally, we consider $0 \le K_p < K_d$. Then the departure angles of the two poles at the origin are still ± 90 deg, and center of the asymptotes lies on the negative real axis, which implies that $0 \le K_p < K_d$ now stabilizes the closed loop system. For example, the root locus with $K_p = 0.5K_d$ looks like the following.



Therefore the only case that stabilizes the closed loop system is $0 \le K_p < K_d$ with $K_d > 0$.

b) The set of all stabilizing controllers lies on the shadowed region below.

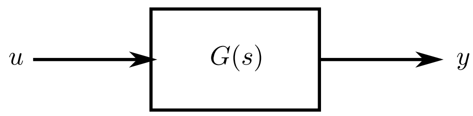


3) *Bode plots of a time delay system (10 points).* Although the course focused on working on linear systems, some of the tools that we studied in class can still be applicable to nonlinear systems analysis.

Consider a nonlinear system that defines the time delay of τ, that is, your output is the copy of your input delayed by τ seconds.

$$y(t) = u(t - \tau)$$

For your information, the block diagram with the Laplace transform is given below.



$$G(s) = \frac{Y(s)}{U(s)} = e^{-\tau s}$$

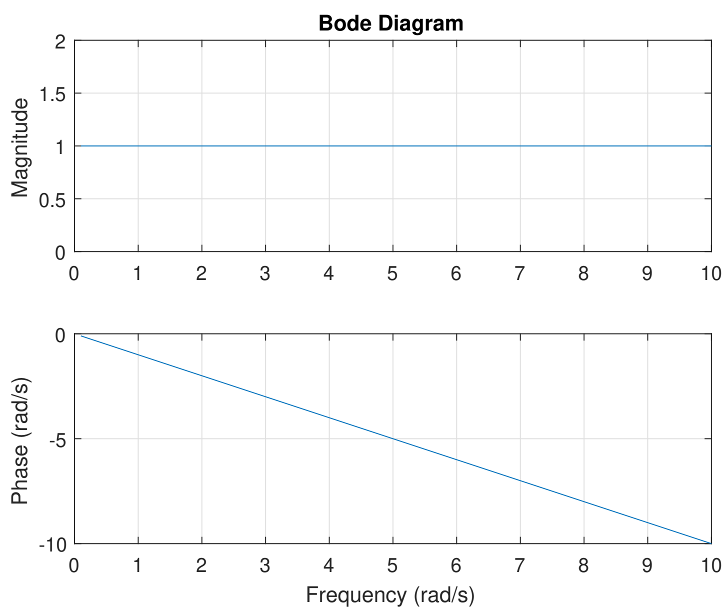
a) Consider $u(t) = \sin \omega t$. What is the output signal $y(t)$? What is the magnitude amplification and the phase delay of $y(t)$?

b) Based on your observations, draw the Bode magnitude and the phase plot of $G(s)$.

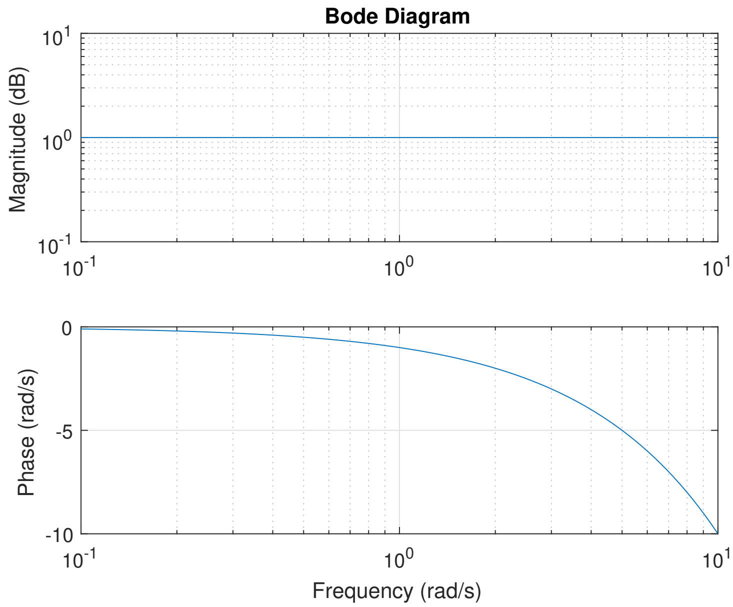
Solution:

a) We have that $y(t) = \sin \omega \,(t - \tau) = \sin\,(\omega t - \omega\tau)$. Hence the magnitude amplification is 1 and the phase delay is $\omega\tau$.

b) The Bode magnitude and the phase plot, for example with $\tau = 1$, should look like



in linear scale, or


Bode Diagram
Magnitude (dB)
10^1
10^0
10^{-1}
10^{-1}
10^0
10^1
Phase (rad/s)
0
-5
-10
10^{-1}
10^0
10^1
Frequency (rad/s)

in log scale.

4) *Small systems analysis (10 points).* A convenient way of describing the *size* of a transfer function is to define a norm. The H_∞ norm of a stable transfer function $G(s)$ is defined as below.

$$||G(s)||_\infty = \sup_{\omega \in \mathbb{R}} |G(jw)|$$

You may not be familiar with the supremum operator (sup), which is ok. The supremum of a signal gives the least upper bound of the signal, for example,

$$\sup_{x \in \mathbb{R}} \left(1 - e^{-x}\right) = 1$$

or

$$\sup_{x \in \mathbb{R}} \left(-\frac{1}{x^2}\right) = 0$$

Hence the supremum operator (sup) is *roughly* equal to the maximum operator (max).

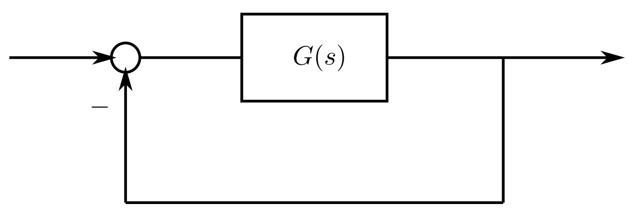
For now, you can simply understand that it implies the maximum of something. We will just say

$$||G(s)||_\infty = \max_{\omega \in \mathbb{R}} |G(jw)|$$

Now the problem. You are given a stable transfer function $G(s)$, whose H_∞ norm is strictly less than 1,

$$||G(s)||_\infty < 1$$

and consider a unity negative feedback loop around $G(s)$ as follows.



Show that the closed loop system is stable.

Solution: Since $||G(s)||_\infty < 1$ implies that $|G(jw)| < 1$ for all $\omega \in \mathbb{R}$, the Nyquist diagram of $G(s)$ never encircles -1, *i.e.*, $N = 0$. Also, $G(s)$ being stable implies that $G(s)$ has no pole on the right half plane, *i.e.*, $P = 0$. The Nyquist stability criterion states that the number of the right half plane poles of the closed loop system, Z, is equal to $Z = N + P = 0$, hence we can conclude that the closed loop system is stable.

5) *Final statistics.*